# REPORT DOCUMENTATION PAGE

*Form Approved*
*OMB No. 0704-0188*



| 1. REPORT DATE | 2. REPORT TYPE | 3. DATES COVERED *(From - To)* |
|---|---|---|
| 25 April 2017 | Briefing Charts | 01 April 2017 - 30 April 2017 |

**4. TITLE AND SUBTITLE**
Multi-scale and multi-physics simulations using the multi-fluid plasma model

**5a. CONTRACT NUMBER**

**5b. GRANT NUMBER**

**5c. PROGRAM ELEMENT NUMBER**

**6. AUTHOR(S)**
Eder Sousa

**5d. PROJECT NUMBER**

**5e. TASK NUMBER**

**5f. WORK UNIT NUMBER**
Q1AM

**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**
Air Force Research Laboratory (AFMC)
AFRL/RQRS
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Edwards AFB, CA 93524-7013

**8. PERFORMING ORGANIZATION REPORT NUMBER**

**9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**
Air Force Research Laboratory (AFMC)
AFRL/RQR
4 Draco Drive
Edwards AFB, CA 93524-7160

**10. SPONSOR/MONITOR'S ACRONYM(S)**

**11. SPONSOR/MONITOR'S REPORT NUMBER(S)**
AFRL-RQ-ED-VG-2017-071

**12. DISTRIBUTION/AVAILABILITY STATEMENT**
Approved for Public Release; Distribution Unlimited. PA Clearance Number: 17211 Clearance Date: 24 April 2017

**13. SUPPLEMENTARY NOTES**
For presentation at Dartmouth University for an Invited Lecture
Prepared in collaboration with ERC. The U.S. Government is joint author of the work and has the right to use, modify, reproduce, release, perform, display, or disclose the work.

**14. ABSTRACT**
Viewgraph/Briefing Charts

**15. SUBJECT TERMS**
N/A

| 16. SECURITY CLASSIFICATION OF: | | | 17. LIMITATION OF ABSTRACT | 18. NUMBER OF PAGES | 19a. NAME OF RESPONSIBLE PERSON |
|---|---|---|---|---|---|
| a. REPORT | b. ABSTRACT | c. THIS PAGE | | | Robert Martin |
| Unclassified | Unclassified | Unclassified | SAR | 31 | 19b. TELEPHONE NUMBER *(Include area code)* N/A |

# Multi-scale and Multi-physics simulations using the Multi-fluid Plasma Model

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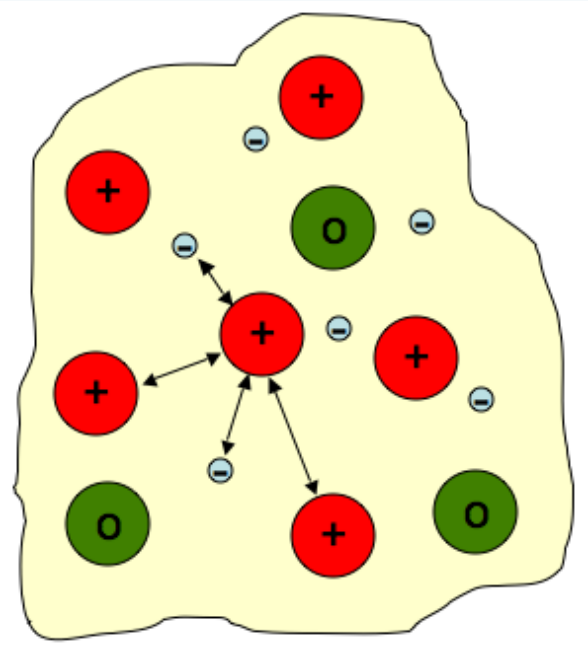
UMass Dartmouth April 25th, 2017

# Outline

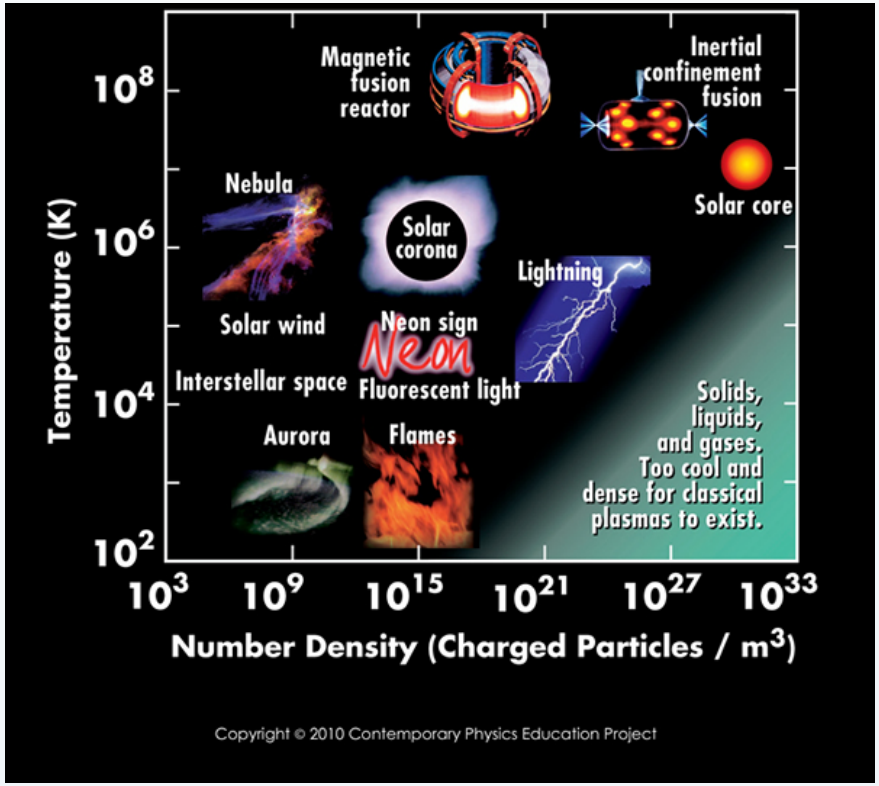
1. Plasma
2. Multi-Fluid Plasma Model
   - Advantages
   - Limits of the model
3. Numerics: Blended Finite Element Method
   - Discontinuous Galerkin for ion/neutrals
   - Continuous Galerkin for electrons/fields
   - Initial tests

# WHAT ARE PLASMAS?

Plasma is a quasineutral gas of charged and neutral particles which exhibits collective behavior.

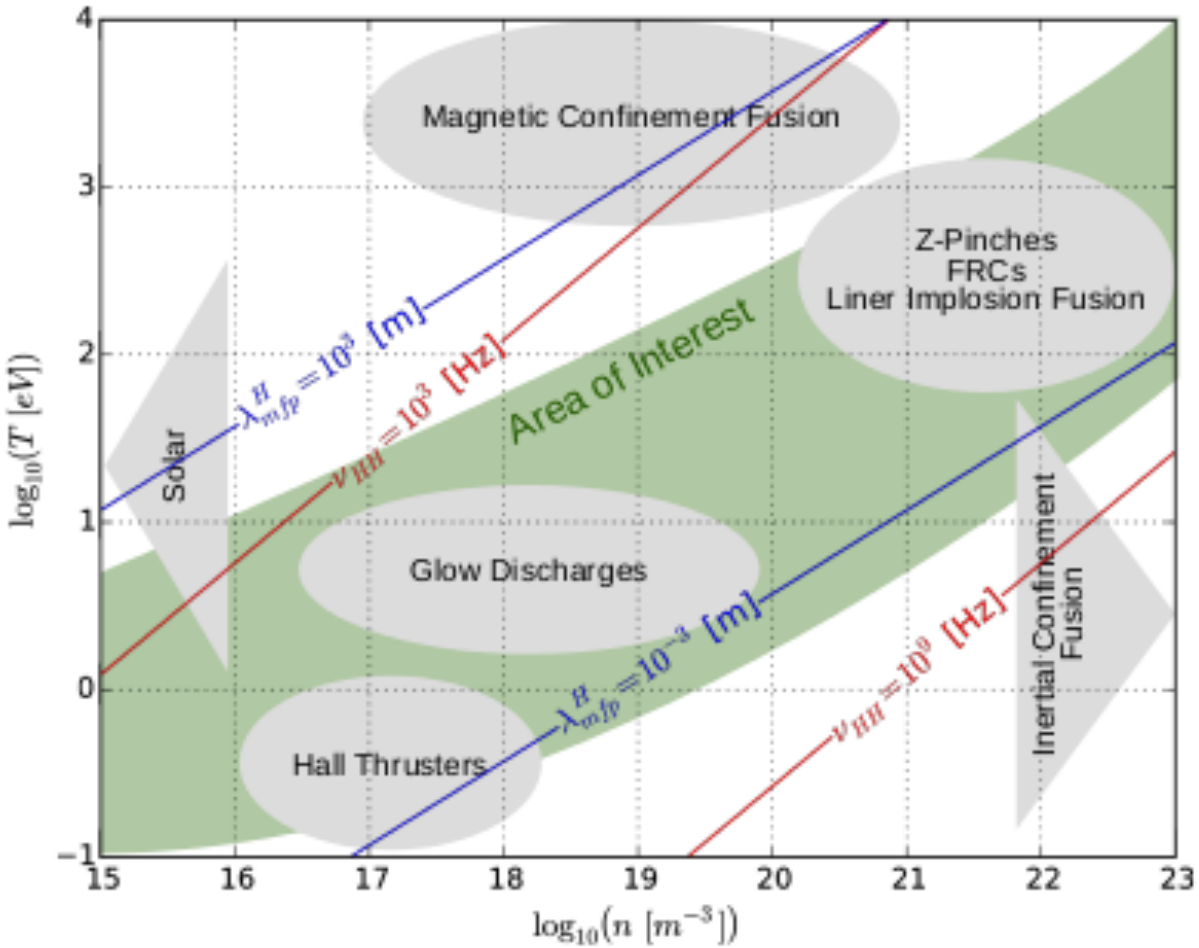


Credit: particleincell.com



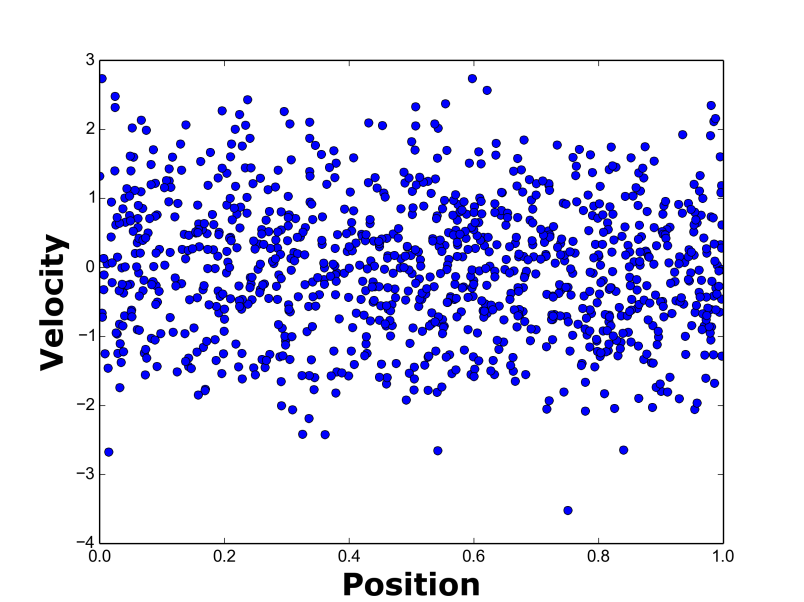

- "99% of matter in the universe is in the state of plasma"

# TIME/SPATIAL SCALES.

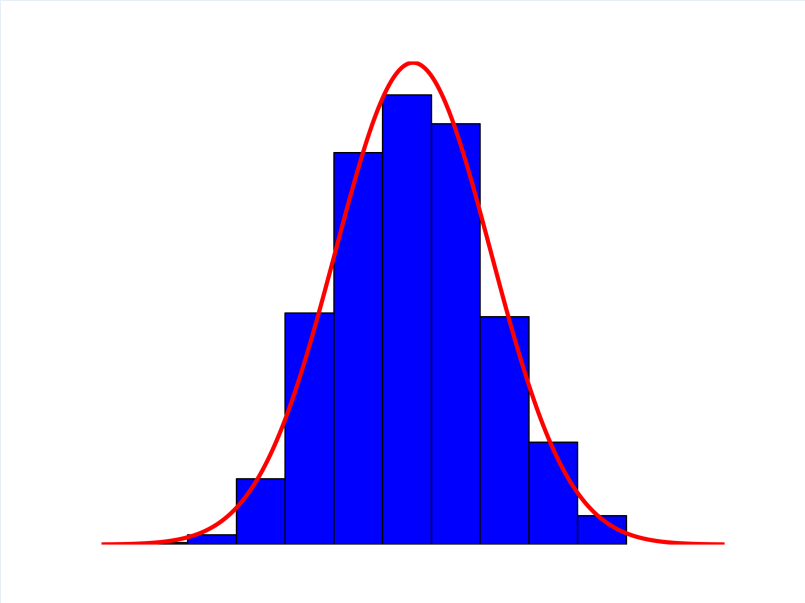



Sean Miller, PhD dissertation, University of Washington (2016)

# There are multiple plasma models.




- 3-Dimensions + 3-Velocities
- Evolve the particles position and velocity
- e.g. Particle-In-Cell models



- Ensemble average of particles distribution, $f_s(\mathbf{x},\mathbf{v},t)$
- Evolve the distribution function
- e.g. Vlasov-Maxwell models

# BOLTZMANN EQUATION EVOLES $f_s$.

- The Boltzmann eqn:

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{q_s}{m_s} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = \left. \frac{\partial f_s}{\partial t} \right|_c$$

- Take the $0^{th}$, $1^{st}$, $2^{nd}$ moments of the Boltzmann Eqn.

$$m_s \int \mathbf{v}^n \frac{\partial f_s}{\partial t} d\mathbf{v} + m_s \int \mathbf{v}^{n+1} \cdot \frac{\partial f_s}{\partial \mathbf{x}} d\mathbf{v} + q_s \int \mathbf{v}^n (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} d\mathbf{v} = m_s \int \mathbf{v}^n \left. \frac{\partial f_s}{\partial t} \right|_c d\mathbf{v}$$

- Each moment of the Boltzmann eqn gives an equation for the moment variable, and introduces the next higher moment variable
- This process can go on indefinitely

# BOLTZMANN EQUATION EVOLES $f_s$.

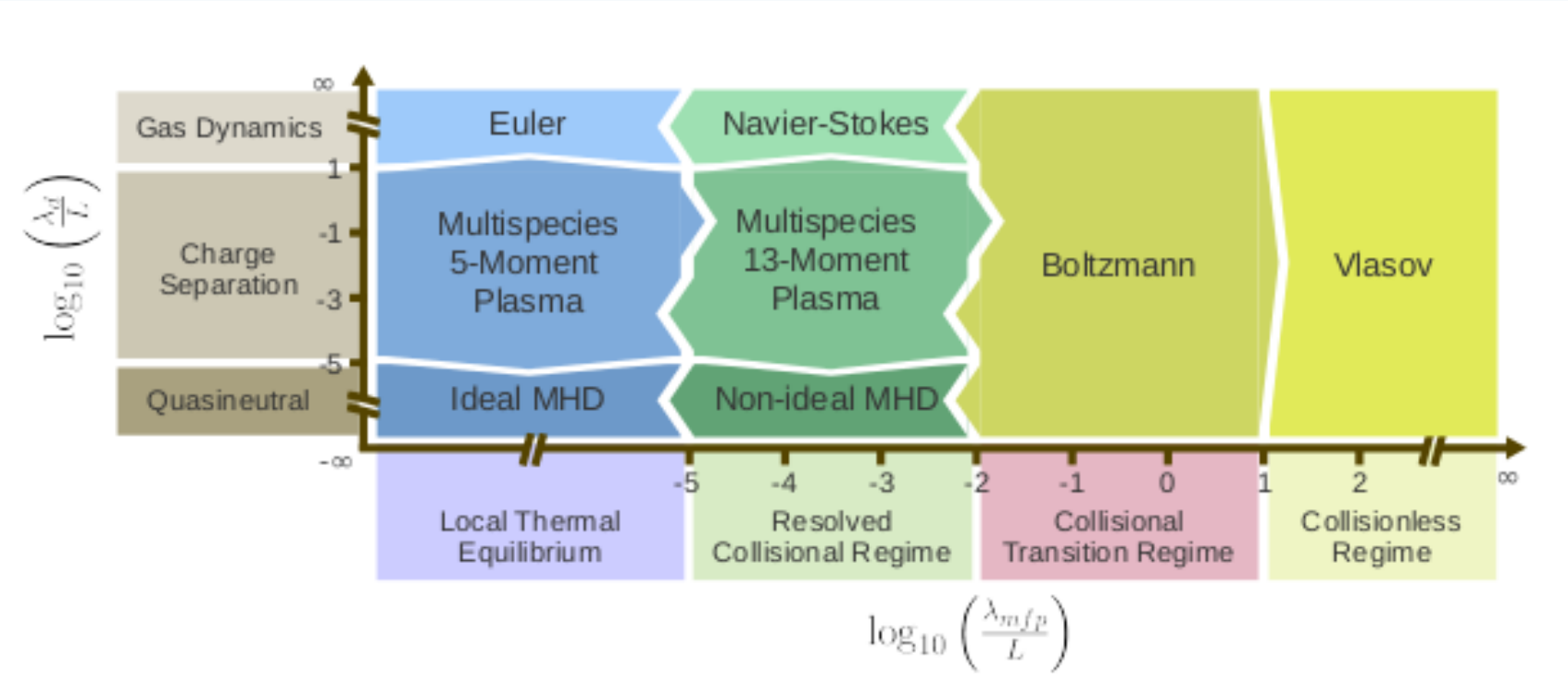
$$\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s) = \left.\frac{\partial \rho_s}{\partial t}\right|_\Gamma$$

$$\frac{\partial \rho_s \mathbf{u}_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s \mathbf{u}_s + p_s \mathbf{I} + \Pi_s) = \frac{\rho_s q_s}{m_s} (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) - \sum_{s*} \mathbf{R}_{s,s*} + \left.\frac{\partial \rho_s \mathbf{u}_s}{\partial t}\right|_\Gamma$$

$$\frac{\partial \varepsilon_s}{\partial t} + \nabla \cdot (((\varepsilon_s + p_s)\mathbf{I} + \Pi_s) \cdot \mathbf{u}_s + \mathbf{h}_s) = \frac{\rho_s q_s}{m_s} \mathbf{u}_s \cdot \mathbf{E} + \sum_{s*} Q_{s,s*} + \left.\frac{\partial \varepsilon_s}{\partial t}\right|_\Gamma$$

- System is truncated by relating higher moment variables to the lower ones
- The fluids are coupled to each other and to the electromagnetic fields through Maxwell's equations and interaction source terms.

# PLASMA MODELS RANGE OF APPLICABILITY.




Sean Miller, PhD dissertation, University of Washington (2016)

# Advantages of the Model

**Kinetic** $\xrightarrow{\text{LTE, velocity moments}}$ **MFPM** $\xrightarrow{\epsilon_o \to 0, \quad m_e \to 0}$ **MHD**

## Ideal MHD model is valid when:

- High collisionality, $\tau_{ii}/\tau \ll 1$
- Small Larmor radius, $r_{Li}/L \ll 1$
- Low Resistivity, $\left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{r_{Li}}{L}\right)^2 \frac{\tau}{\tau_{ii}} \ll 1$

## Multi-Fluid Plasma Model

- Less computationally expensive than kinetic models
- Multi-fluid effects become relevant at small spacial and temporal scales
- Finite electron mass and speed-of-light effects are included
- There is charge separation is modeled
- Displacement current effects are resolved in the MFPM

# THE MFPM HAS DISPERSIVE SOURCES.

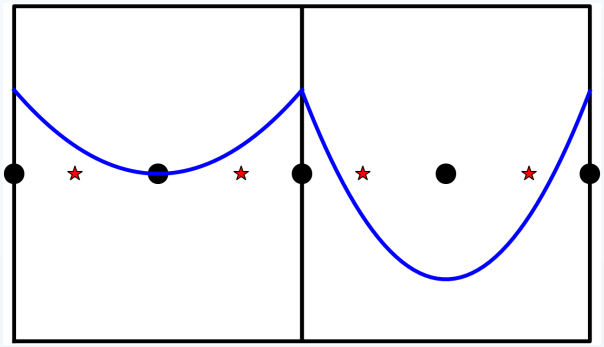
$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \overleftrightarrow{\mathbf{F}}}{\partial \mathbf{x}} = \mathbf{S}$$

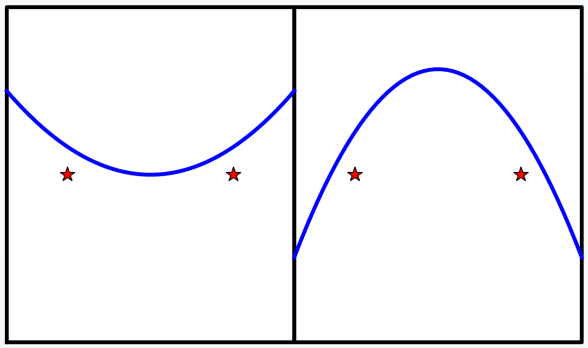
- The source Jacobian $\frac{\partial \mathbf{S}}{\partial \mathbf{Q}}$ has imaginary eigenvalues
- The equation system has dispersive sources
- The dispersion is physical (may be difficult to distinguish from numerical dispersion)
- This dispersion is due to plasma waves that result from ion and electron plasma interactions with electromagnetic fields
- An ideal numerical method for the MFPM should:
  - be high-order accurate
  - capture shocks
  - couple the flux and the sources
  - not impose strict time-step

Srinivasan et al, CCP 10 (2011)

# BFEM SIMULTANEOUSLY USES CG AND DG.

- Solution to the electron and EM fields is smooth and does not shock





- Continuous Galerkin
- Electron fluid and EM fields

$$\mathbf{Q} = \sum_i \mathbf{q}_i v_i$$

- Discontinuous Galerkin
- Multiple ion and neutral fluids

$$\mathbf{Q} = \sum_i \mathbf{c}_i v_i$$

- For this implementation the balance law form is cast as

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \overleftrightarrow{\mathbf{F}}}{\partial \mathbf{Q}} \cdot \frac{\partial \mathbf{Q}}{\partial \mathbf{x}} = \mathbf{S} + \kappa \nabla^2 \mathbf{Q}_d$$

- Lagrange polynomials are used for basis functions, $v_r$

$$\int_\Omega v_r \frac{\partial \mathbf{Q}}{\partial t} dV = \mathcal{L}_r(\mathbf{Q}) = \int_\Omega v_r \mathbf{S} dV - \int_\Omega v_r \frac{\partial \overleftrightarrow{\mathbf{F}}}{\partial \mathbf{Q}} \cdot \frac{\partial \mathbf{Q}}{\partial \mathbf{x}} dV + \kappa \int_\Omega v_r \nabla^2 \mathbf{Q}_d dV$$

- $\theta$-method time integration

$$\mathcal{R}(\mathbf{Q}^n) = \overleftrightarrow{\mathbf{M}} \frac{\mathbf{Q}^{n+1} - \mathbf{Q}^n}{dt} - \theta \mathcal{L}_r(\mathbf{Q}^{n+1}) - (1-\theta)\mathcal{L}_r(\mathbf{Q}^n) = 0$$

- $\theta = 0.5$ is used for $2^{nd}$ order accuracy

$$\overleftrightarrow{\mathbf{J}}(\mathbf{Q}^n) = \frac{\partial \mathcal{R}(\mathbf{Q}^n)}{\partial \mathbf{Q}^n}, \quad \overleftrightarrow{\mathbf{J}}(\mathbf{Q}^n)\Delta\mathbf{Q} = -\mathcal{R}(\mathbf{Q}^n), \quad \mathbf{Q}^{n+1} = \mathbf{Q}^n + \Delta\mathbf{Q}$$

Reddy, An Introduction to the Finite Element Method (2006)

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \overleftrightarrow{\mathbf{F}}}{\partial \mathbf{x}} = \mathbf{S}$$

- Legendre polynomials are used for basis functions, $v_p$
- The hyperbolic equation is multiplied by the basis function,

$$\int_\Omega v_p \frac{\partial \mathbf{Q}}{\partial t} dV = \mathcal{L}_p(\mathbf{Q}) = \int_\Omega v_p \mathbf{S} dV - \oint_{\partial\Omega} v_p \overleftrightarrow{\mathbf{F}} \cdot d\mathbf{A} + \int_\Omega \overleftrightarrow{\mathbf{F}} \cdot \nabla v_p dV$$

- Explicit Runge-Kutta time integration
- $CFL = c\Delta t/\Delta x \leq 1/(2p-1)$, $p$ is the polynomial order

$$\mathbf{Q}^* = \mathbf{Q}^n + \Delta t \mathcal{L}_p(\mathbf{Q}^n),$$

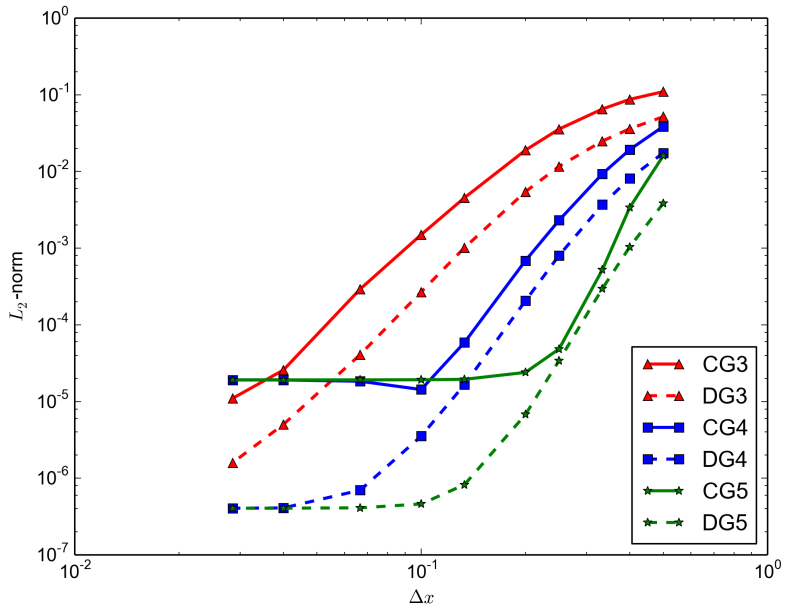
$$\mathbf{Q}^{n+1} = \frac{1}{2}\mathbf{Q}^* + \frac{1}{2}\mathbf{Q}^n + \frac{1}{2}\Delta t \mathcal{L}_p(\mathbf{Q}^*).$$

Loverich et al, CCP 9 (2006)

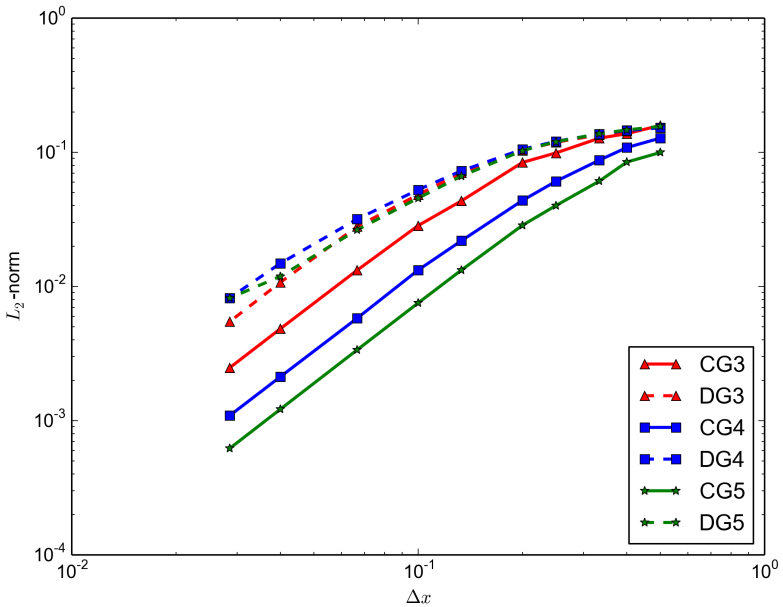
# Convergence of the BFEM.

$$\frac{\partial Q}{\partial t} + \frac{\partial Q}{\partial x} = 0, \qquad Q(x,0) = e^{-10(x-8)^2}, \qquad ||\Delta Q||_2 = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\hat{Q} - Q_i)^2}$$

## Spatial Convergence



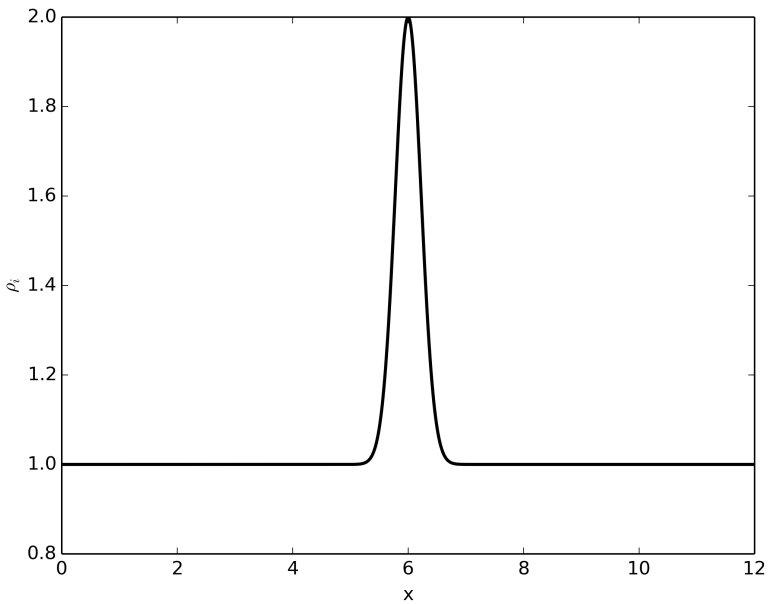

## Temporal Convergence




- Simulations at fixed time-step
- Simulations at fixed CFL=1

# 1D Soliton Problem

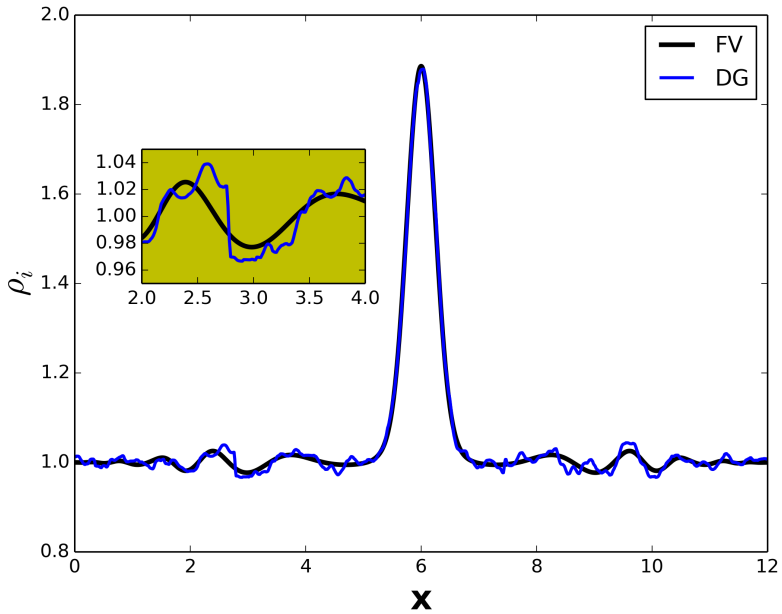
- 1D soliton is a two-fluid plasma problem
- The solution is smooth, therefore artificial dissipation can be small
- The simulation uses 512 second-order elements
- $B_z = 1.0$, $T_e = T_i = 0.01$, $\mathbf{u}_i = \mathbf{u}_e = \mathbf{0}$
- $n_e = n_i = 1.0 + e^{-10(x-6)^2}$

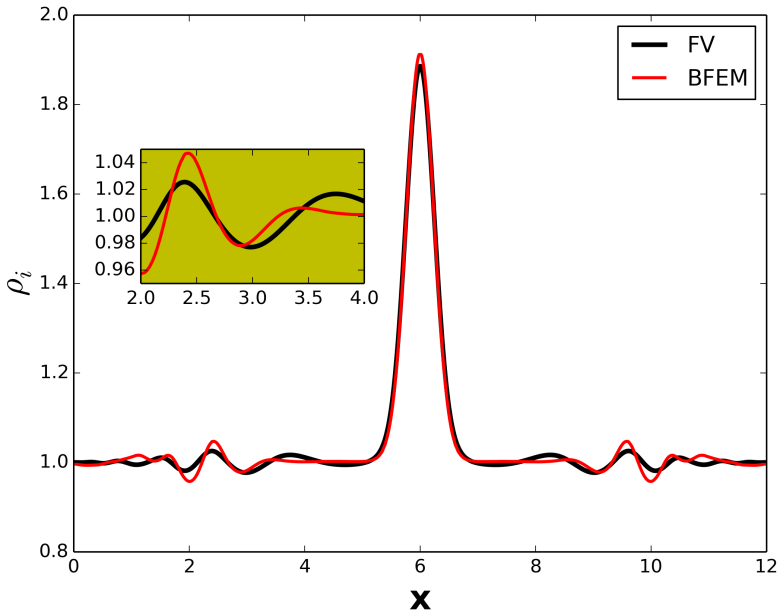



Baboolal, Math. and Comp. Sim. 55 (2001)

# DG AND BFEM COMPARISON WITH A SOLUTION

$$\frac{m_i}{m_e} = 1836, \frac{c}{c_{si}} = 1000\sqrt{2}, \text{FV 5000 cells}$$




- DG Solution is very dispersive
- BFEM is less dissipative than the converged solution

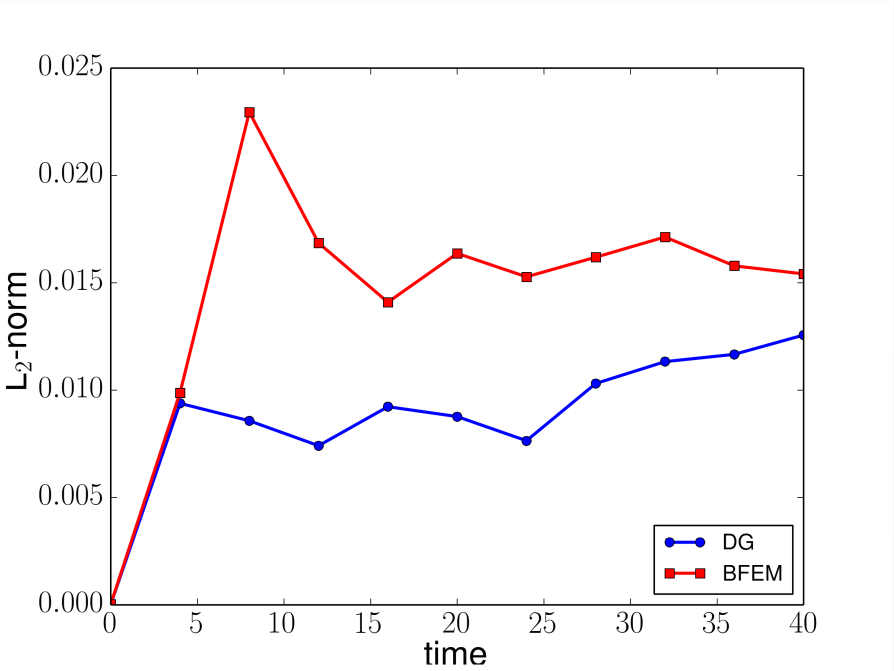
Hakim et al, JCP 219 (2006)

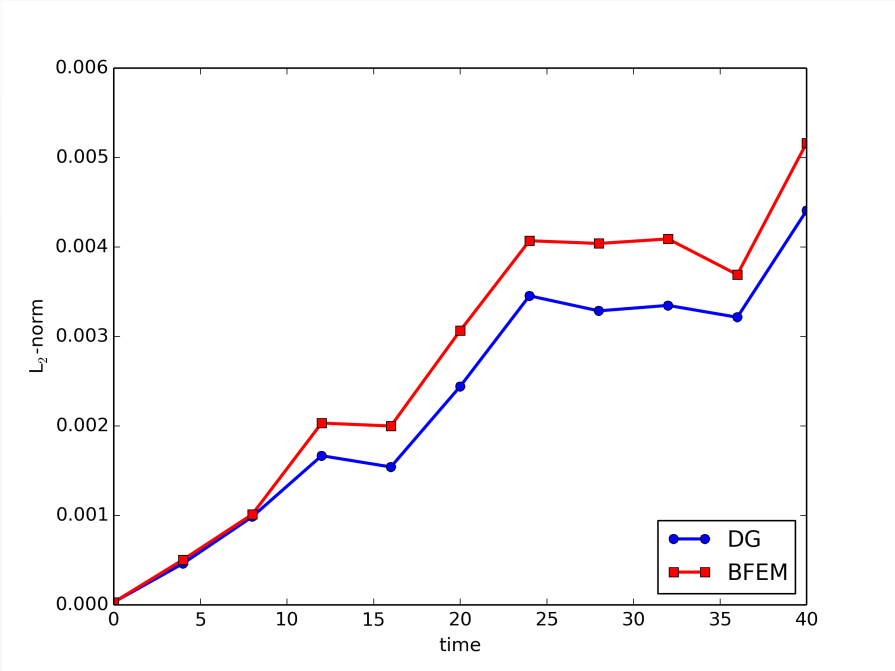
# BFEM Computational Cost Savings

| case | $m_i/m_e$ | $c/c_{si}$ | DG time(s) | BFEM time (s) | BFEM cost over DG |
|---|---|---|---|---|---|
| 1 | 25 | $10/\sqrt{2}$ | 0.32 | 37.7 | +11681% |
| 2 | 100 | $10/\sqrt{2}$ | 1.28 | 37.7 | +2845% |
| 3 | 500 | $10/\sqrt{2}$ | 6.82 | 37.7 | +452.8% |
| 4 | 1000 | $10/\sqrt{2}$ | 12.4 | 38.2 | +208.1% |
| 5 | 1836 | $10/\sqrt{2}$ | 23.5 | 40.4 | +71.91% |
| 6 | 3672 | $10/\sqrt{2}$ | 47.2 | 39.2 | -16.95% |
| 7 | 3672 | $100/\sqrt{2}$ | 520 | 265 | -49.04% |
| 8 | 3672 | $1000/\sqrt{2}$ | 5274 | 2735 | -48.14% |

- Per time-step explicit DG is faster than BFEM, but it requires many more time-steps
- BFEM is more efficient only when time-step are considerably larger than explicit DG

# BFEM Accuracy



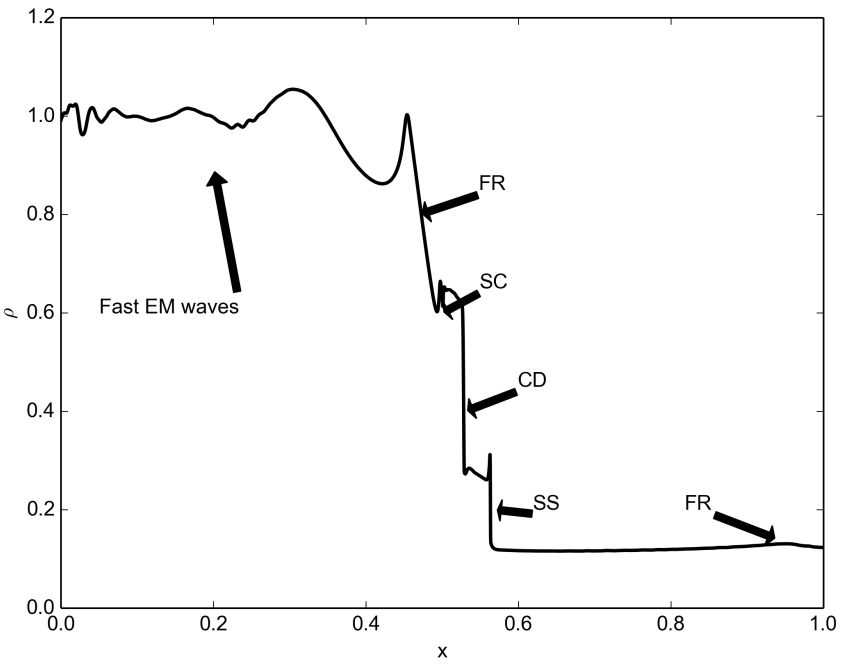




$m_i/m_e = 1836$ $m_i/m_e = 1$

- The BFEM seems to be less accurate than the DG implementation ($\sim 50\%$)
- When the mass ratio is one, the two methods have the same level of accuracy
- The discrepancy is due to the fact that the semi-implicit BFEM does not resolve the plasma frequency in this problem

# Electromagnetic Plasma Shock Problem

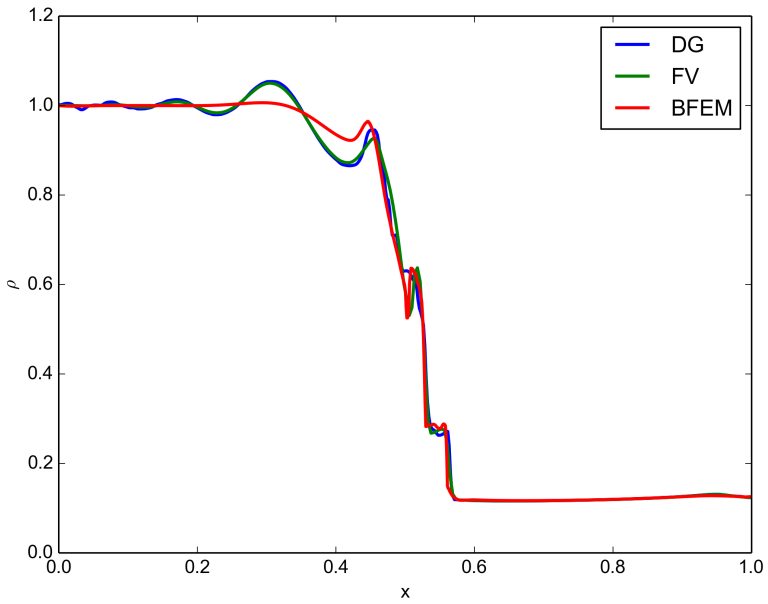
- Fast rarefaction wave (FR), a slow compound wave (SC), a contact discontinuity (CD), a slow shock (SS), and another fast rarefaction wave (FR)
- The problem exhibits limits of MHD and multi-fluid behavior by changing the Larmor radius, $r_L$
  - MHD: $r_L \rightarrow 0$
  - Multi-fluid: $r_L \sim L$



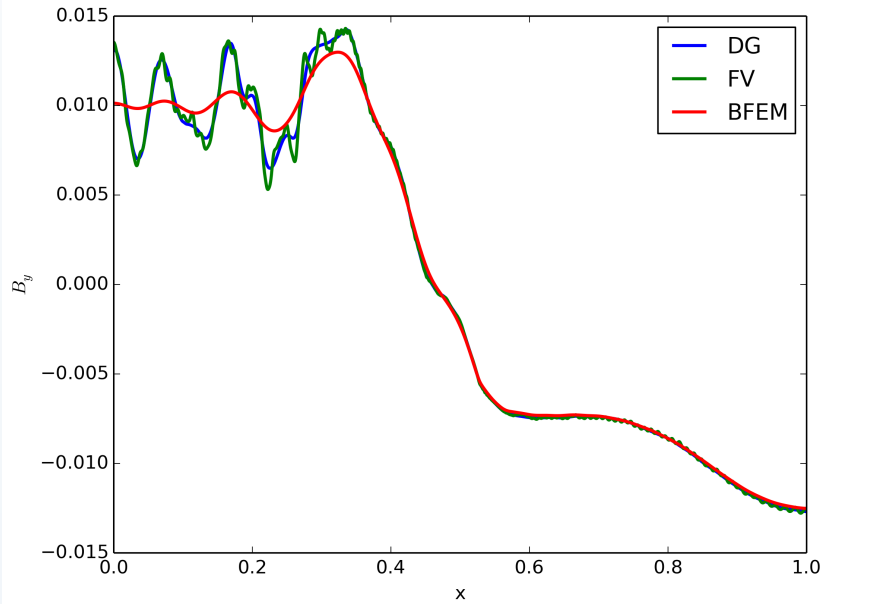

Brio and Wu, JCP 75 (1988)

# Shock in Density but Smooth Fields.

- t=0.05/$\omega_{ci}$, c/$c_{si}$=110, $m_i/m_e$=1836




mass density



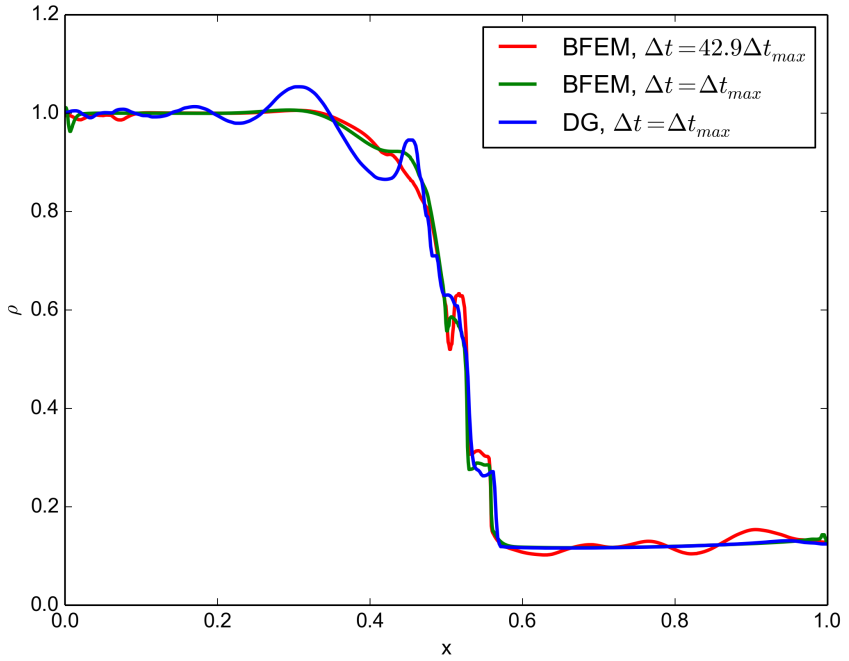

Magnetic field (y-comp.)

- The main features of the problem are captured by all three methods
- BFEM does not properly resolve the fast electromagnetic waves which require accurately resolving the electron dynamics

# Maximum BFEM time-step

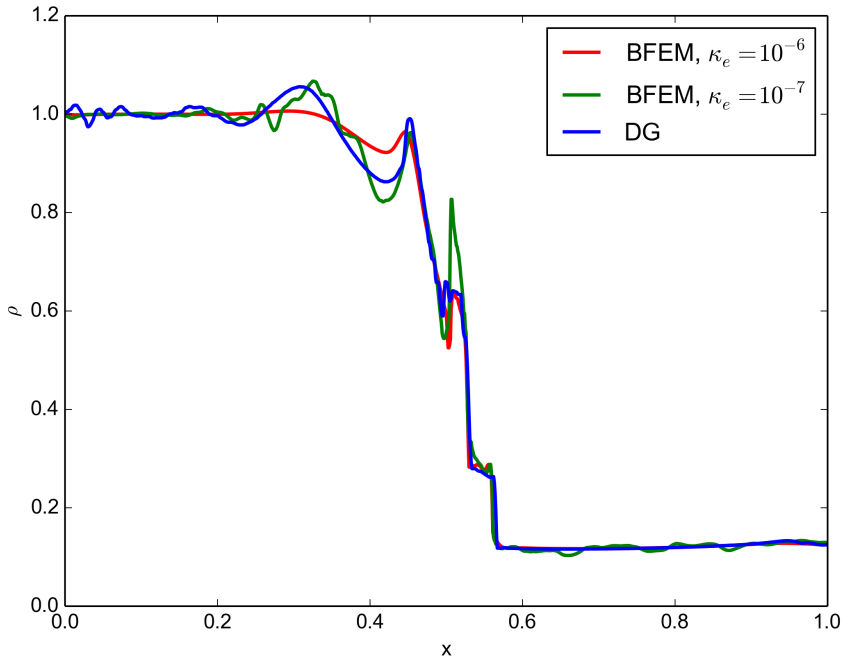
$$\Delta t_{max} = \min\left(\frac{\Delta x}{c_{se}}, \frac{\Delta x}{c_{si}}, \frac{\Delta x}{c}, \frac{0.1}{\omega_{ce}}, \frac{0.1}{\omega_{ci}}, \frac{0.1}{\omega_{pe}}, \frac{0.1}{\omega_{pi}}\right)$$

- $\Delta t_{max}$ corresponds to the maximum value allowed for explicit methods based on the CFL condition
- $\Delta t = 42.9\Delta t_{max}$ is the maximum time step allowed by the BFEM due to ion dynamics



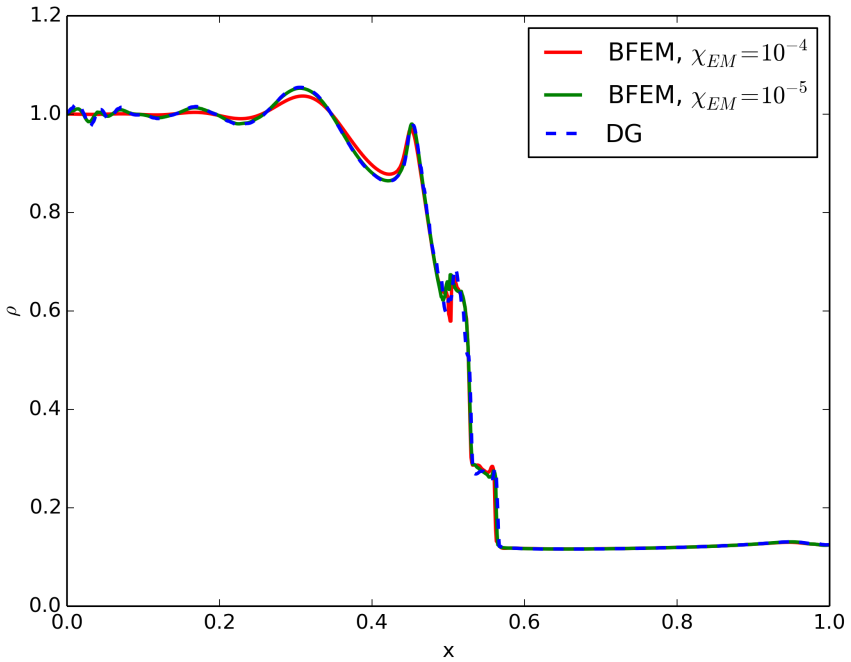
# Effects of Artificial Dissipation

- Varying the artificial dissipation on the electron fluid, $\kappa_e$
- Wave-like behavior of the problem is better resolved
- Amplitude of the compound wave increases
- Right fast rarefaction wave is not visible



# Effects of Artificial Dissipation

- Varying the artificial dissipation on the EM-field, $\kappa_{EM}$
- There is better agreement with the DG solution
- This reinforces the point that the wave-like behavior arises from the interaction of the electron fluid with the electromagnetic fields



# Summary

- The blended finite element method (BFEM) is presented
  - DG spatial discretization with explicit Runge-Kutta ($i^+$, n)
  - CG spatial discretization with implicit Crank-Nicolson ($e^-$, fileds)
  - DG captures shocks and discontinuities
  - CG is efficient and robust for smooth solutions
- Physics-based decomposition of the algorithm yields numerical solutions that resolve the desired timescales
- DG method takes less computational time to advance the solution by one time-step, however $\Delta t$ is much smaller than that of the BFEM
- Computational cost savings using the BFEM will only occur for relatively large implicit time-steps compared to explicit time-steps

Sousa and Shumlak, JCP 326 (2016) 56-75

Thank you.

# EXTRAS

# Physical description of a fluid.

- Modeling each particle velocity and position is not practical.
- Instead an average is performed to give a statistical description.
- Calculate the number of particles per unit volume having approximately the velocity $\mathbf{v}$ near the position $\mathbf{x}$ and at time $t$, distribution function $f(\mathbf{v}, \mathbf{x}, \mathbf{t})$

$$\rho_s = m_s \int f_s(\mathbf{v}) d\mathbf{v}$$

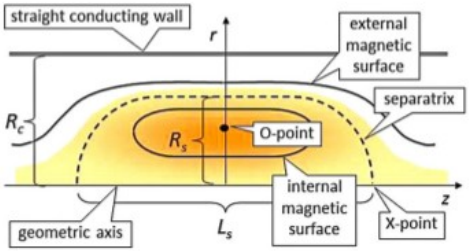
$$\rho_s \mathbf{u}_s = m_s \int \mathbf{v} f_s(\mathbf{v}) d\mathbf{v}$$

$$\mathbb{P}_s = \mathbf{P}_s = m_s \int \mathbf{ww} f_s(\mathbf{v}) d\mathbf{v}, \quad p_s = \frac{1}{3} m_s \int w^2 f_s(\mathbf{v}) d\mathbf{v}$$

$$\mathbf{H}_s = m_s \int \mathbf{www} f_s(\mathbf{v}) d\mathbf{v}, \quad \mathbf{h}_s = \frac{1}{2} m_s \int w^2 \mathbf{w} f_s(\mathbf{v}) d\mathbf{v}$$

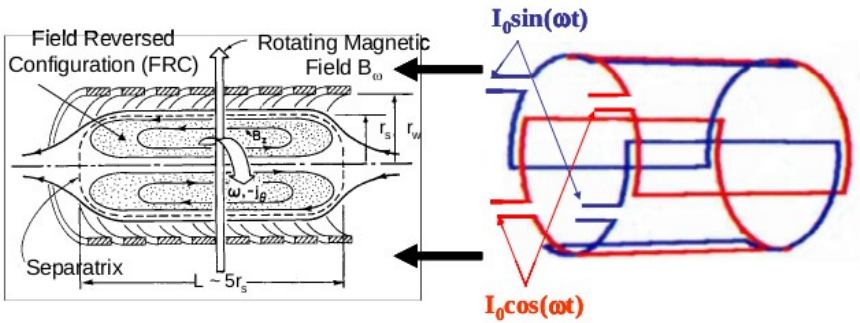
$$\mathbf{w} = \mathbf{v} - \mathbf{u}_s$$

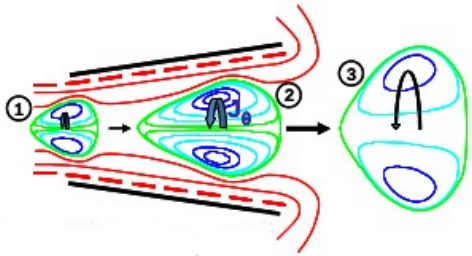
# Field Reversed Configuration Thruster




- Compact toroid, no central column
- Simple geometry and B-field configuration
- High power density (high plasma beta)
- Highly movable

- A Rotating Magnetic Field (RMF) is used to form the FRC
- Strong magnetization of the electrons to the RMF produces a current
- To conserve the total flux the B-field at the center reverses



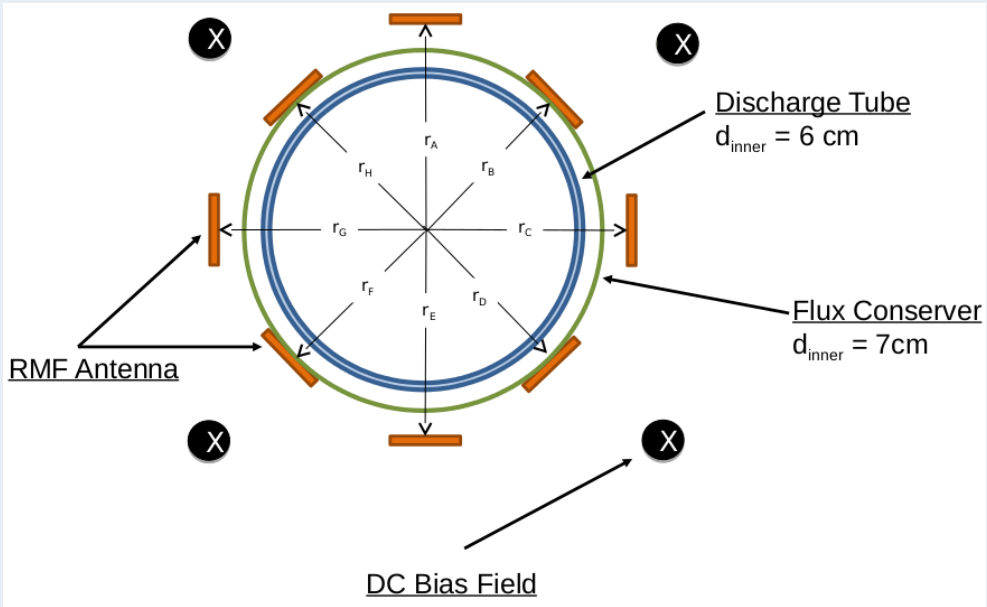

- 1 - Formation of high density FRC
- 2 - Acceleration of the FRC by Lorentz force
- 3 - FRC expands, converting thermal energy to directed energy

**Advantages:**

- Electrodeless and the plasmoid propellant is magnetically isolated from the walls
- Propellant is completely uncoupled from the driving and confining fields
- High plasma temperatures and densities significantly reduce ionization losses

# FRC FORMATION STUDIES.

- The formation of FRC using a rotating magnetic field (RMF)
- $\omega_{ci} < \omega < \omega_{ce}$
- $\nu_{ei} \ll \omega_{ce}$




- Flux Conserver BC

$$B_z(a,t) = \frac{b^2 B_a(0)}{b^2 - a^2} - \frac{1}{\pi(b^2 - a^2)} \int_0^a r dr \int_0^{2\pi} d\theta B_z(r,\theta,t)$$

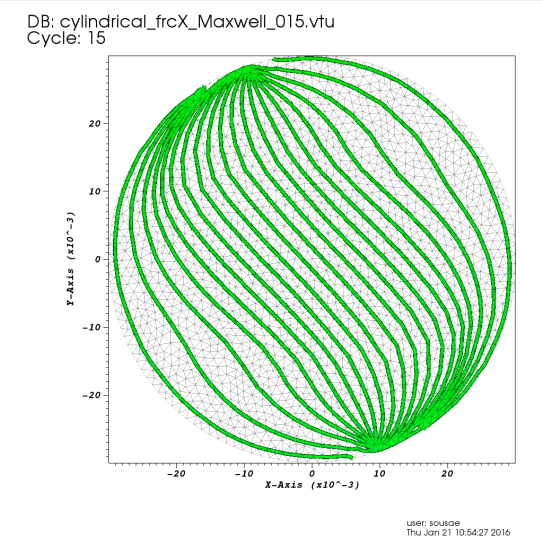
- RMF BC

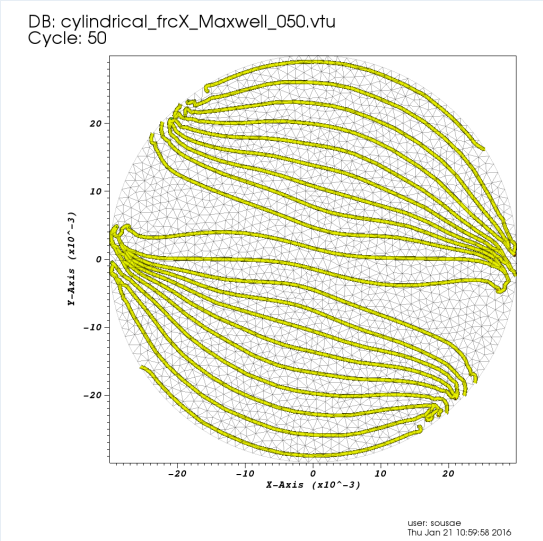
$$I_{RMF} = I_t \cos(\omega t + \phi)$$
$$I_t = I_o(1 - e^{-t/\tau})$$

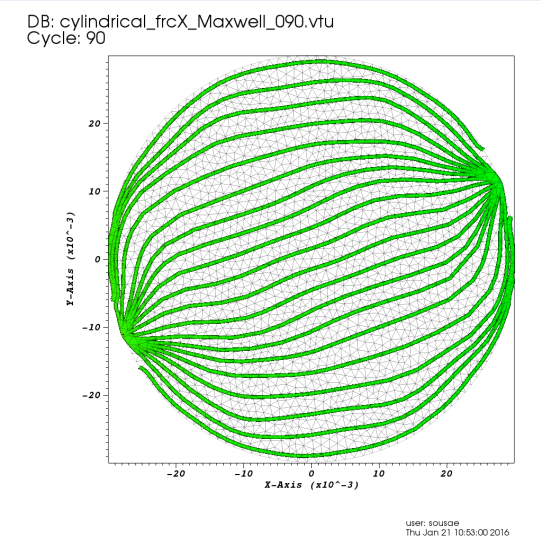
- $\phi_{A,B} = 0, \phi_{E,F} = \pi$
- $\phi_{C,D} = \pi/2,$ $\phi_{G,H} = -\pi/2$
- Bias Field is constant at t=0s

# RMF Antenna Modeling.

- Initialization: $B_\omega$=90G, $B_z$=50G, $\omega$=5MHz
- Solve Maxwell's eqns. with divergence constraints

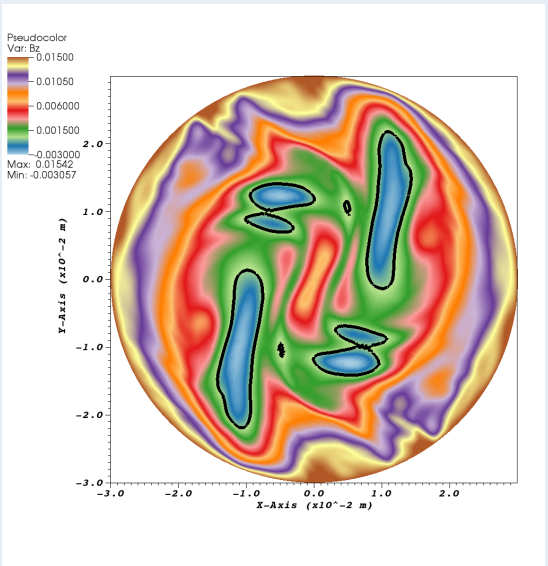


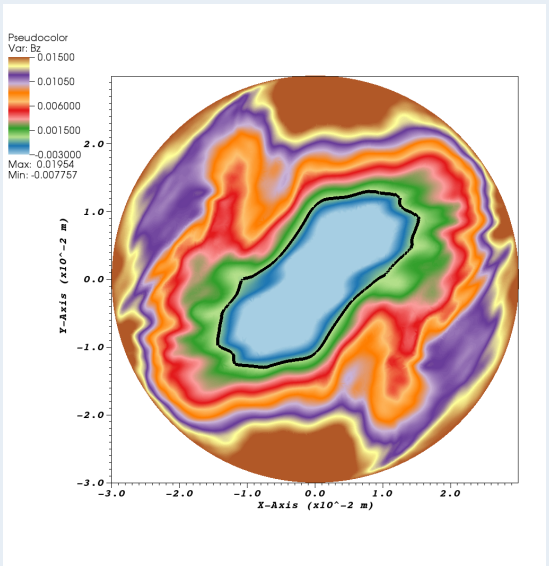



# FRC Formation Simulations.

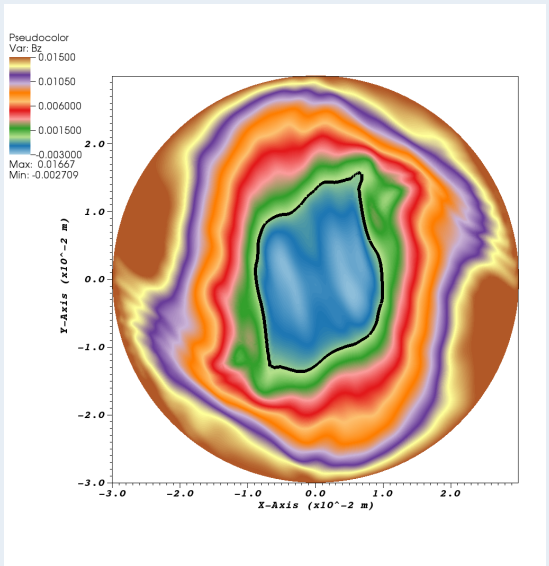
- Initialization: $T_i = T_e = 30\text{eV}$, $n_i = n_e = 10^{19}\text{m}^{-3}$, $m_i/m_e = 1836$
- Solve Multi-Fluid eqns.
- $B_z$ is plotted




$t = 250ns$




$t = 500ns$




$t = 1500ns$

# (Near) Future Work

## Neutral-Plasma Model

- Interaction of multi-fluid plasma with gas dynamics neutral fluid
  - electron-impact ionization

$$e^- + n \rightarrow i^+ + 2e^- - \phi_{ion}$$

  - radiative recombination

$$e^- + i^+ \rightarrow n + h\nu$$

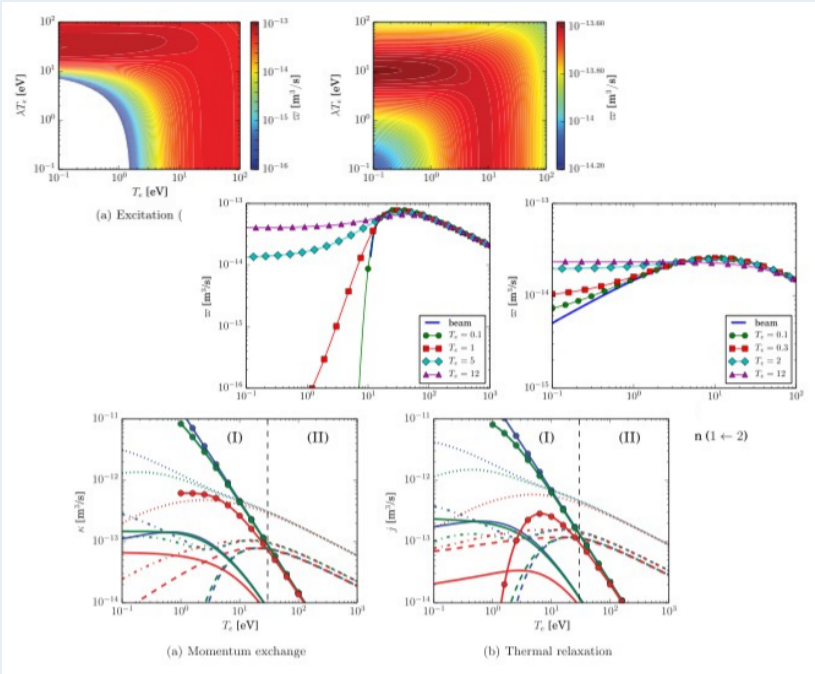
  - resonant charge exchange

$$i^+ + n \rightarrow n + i^+$$

Meier and Shumlak, PoP 19 072508 (2012)

## Collisional-Radiative Model

- Excitation/De-excitation rates




Le and Cambier, PoP 22 093512 (2015)